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Dynamic Efficiency of Pulsed Plasma Accelerators

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In most pulsed plasma accelerators, such as coaxial guns, pinch engines, T-tubes, etc., the essential magnetogasdynamic interaction is the acceleration of a current-carrying region of plasma by its own magnetic field into an ambient body of gas. The piston action of this accelerated current zone in turn generates a strong shock wave that propagates slightly ahead of it and serves to compress the ambient gas as well as accelerate it to the piston velocity. It is well known that such strong shock waves propagating at constant velocity into gases at rest convert about 50% of the available energy into organized streaming motion of the shocked gas and 50% into its internal, i.e., random thermal, energy. This property has on occasion been invoked as a criterion for the limiting performance of such accelerators. For example, Larson et al.[1] have contended that, in the range of shock strengths prevalent in useful pulsed plasma thrustors, the specific enthalpy of the shocked gas is so high that the bulk of its internal energy will be lost by radiation before it can be recovered by expansion at the orifice. Thus, they suggest, such thrustors may labor under an intrinsic 50% efficiency limitation.

This criterion seems excessively severe. On the one hand there is certain experimental evidence that the internal equilibration and radiation processes are not sufficiently rapid to dispose of the bulk of the internal energy on the time scale involved.[2] But, even conceding no recovery of internal energy for useful thrust work, the 50% criterion pertains only to special gasdynamic situations, such as the quoted case of constant velocity propagation into ambient gas at rest. Guman,[3] for example, has shown that a prior streaming motion of the ambient gas fill substantially alters the energy division for constant shock propagation velocity. It is the purpose of this note to display a simple analytical argument whereby the energy division can be estimated for arbitrary profiles of shock velocity along the accelerator channel as well as for arbitrary one-dimensional accelerator geometries and initial gas density distributions.

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Consider the conventional snowplow idealization of the process,[4] wherein a thin current sheet is driven by its own magnetic field into a gas at rest and accumulates within a negligibly narrow region on its surface all of the gas that it overtakes. Starting from a simple Newtonian statement of the over-all dynamics of the situation, one can identify two components of the rate of energy deposition

$$vF = v(d/dt)(mv) = (d/dt)(\tfrac{1}{2}mv^2) + (v^2/2)(dm/dt) \quad (1)$$

where F is the instantaneous driving force, m the mass accumulated on the piston, and v its velocity. The first term on the right is the rate of increase of streaming kinetic energy of the swept gas immediately useful for propulsion. The second term represents the dissipation associated with the inelastic collision of the incoming particles with the snowplow piston. (The rigorous validity of this representation for the actual gasdynamic acceleration process behind a strong shock wave is a rather subtle point developed in detail elsewhere.[5])

Imagine that the preceding process starts with some initial velocity v_0 and continues over a period of time t_f, at the end of which the piston has progressed a distance x_f and has accumulated a total mass m_f, which is now moving at a velocity v_f. The ratio of interest is that of the integrals of the foregoing two terms over this period:

$$\alpha = \left(\int_0^{t_f} \frac{v^2}{2}\frac{dm}{dt}\,dt\right)\bigg/\left(\frac{1}{2}m_f v_f^2\right) \quad (2)$$

which determines the dynamic efficiency of the process

$$\eta = 1/(1+\alpha) \quad (3)$$

At this point it might be noted that, in the very special case where the piston begins its motion with the total mass to be accelerated already entrained ("slug" model), dm/dt will be zero thereafter, and the efficiency η will be in identical unity.

Returning to the general problem, we convert to dimensionless distance, velocity, mass, and time variables:

$$X = x/x_f \qquad V = v/v_0 \qquad M = m/m_f \qquad T = tv_0/x_f \quad (4)$$

The geometry of a specific accelerator and the ambient gas density distribution are embodied in the functional dependence of accumulated mass on piston position

$$M = \mu(X) \quad (5)$$

In terms of these dimensionless quantities, the energy-division ratio α becomes

$$\alpha = \left[\int_0^{T_f}\left(\frac{dX}{dT}\right)^3\frac{d\mu}{dX}\,dT\right]\bigg/\left(\frac{dX}{dT}\right)_f^2 = \left[\int_0^1 V^2\frac{d\mu}{dX}\,dX\right]\bigg/ V_f^2 \quad (6)$$

In general, the development of the dimensionless velocity profile $V(X)$ is determined by the magnitude and waveform of the discharge current as well as by the geometry and ambient gas distribution. The information we desire here, however, may be obtained by examining the behavior of α for various assumed velocity profiles.

Table 1 Dynamic efficiencies for exponential velocity profile

k	α	η
0	1.00	0.50
1	0.43	0.70
2	0.25	0.80
3	0.17	0.85
10	0.05	0.95
∞	0.00	1.00

Table 2 Dynamic efficiencies for power law velocity profile

v_f/v_0	n	α	η
1	all	1.00	0.50
2	1	0.58	0.63
2	2	0.47	0.68
2	4	0.38	0.73
2	∞	0.25	0.80
4	1	0.44	0.70
4	2	0.30	0.77
4	4	0.20	0.83
4	∞	0.06	0.94
∞	1	0.33	0.75
∞	2	0.20	0.83
∞	4	0.11	0.90
∞	∞	0.00	1.00

For example, any constant velocity accelerator is seen to subscribe to the previously mentioned equal partition criterion, since here V has the value 1 throughout, and hence

$$\alpha = \int_0^{\mu_f} d\mu = 1 \tag{7}$$

Note that this result is independent of accelerator geometry and initial gas distribution.

Other velocity profiles must be examined in the light of specified initial mass distributions $\mu(X)$. For all linear accelerators (coaxial guns, parallel plate accelerators, T-tubes, etc.) having uniform initial gas fills, $\mu(X) = X$ and $d\mu/dX$ disappears from the α integral [Eq. (6)]. As a first illustration of these cases, consider the family of exponential velocity profiles $V = e^{kX}$ which yield

$$\alpha = (1/2k)(1 - e^{-2k}) \tag{8}$$

and the corresponding range of efficiencies shown in Table 1.

In a similar way, the general power law profiles $V = 1 + bX^n$ yield

$$\alpha = 1/(1 + b)^2\{1 + [2b/(n + 1)] + [b^2/(2n + 1)]\} \tag{9}$$

where the coefficient b clearly relates the initial and final velocities:

$$v_f/v_0 = 1 + b \tag{10}$$

The efficiencies of these profiles depend on both this velocity ratio and the value of the exponent, as displayed in Table 2.

The effect of a variable channel cross section may be illustrated by the familiar case of the linear pinch, where the position coordinate is now $x = r_0 - r$, and the corresponding dimensionless quantity is $X = 1 - (r/r_0)$, where r_0 refers to the initial current sheet radius. The mass accumulation function for uniform initial density is now $\mu(X) = X(2 - X)$. Returning to the exponential velocity profile $V = e^{kX}$, the evaluation of Eq. (6) now yields

$$\alpha = (1/2k^2)(1 - e^{-2k} - 2ke^{-2k}) \tag{11}$$

with the range of efficiencies shown in Table 3.

Table 3 Dynamic efficiencies for pinch geometry, exponential velocity profile

k	α	η
0	1.000	0.500
1	0.300	0.770
2	0.110	0.900
3	0.050	0.950
10	0.005	0.995
∞	0.000	1.000

The efficiency dealt with here is, of course, strictly concerned with the dynamic processes in the accelerator channel and makes no reference to other possible losses in the complete device. For example, if the velocity profiles discussed previously predicate the passage of relatively large currents through the plasma near the end of the acceleration process, provision must be made to recover the associated magnetic-field energy at the time of ejection lest this loss dominate the problem.

Nevertheless, within the chosen context, the previous calculations demonstrate that the dynamic efficiency of pulsed plasma accelerators does not have a general bound, but is determined by specific details of the piston velocity profile, channel geometry, and ambient gas distribution. In general, this efficiency is seen to be favored by vigorously accelerating velocity profiles, converging accelerator geometries, and decreasing ambient density profiles in the direction of propagation.

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